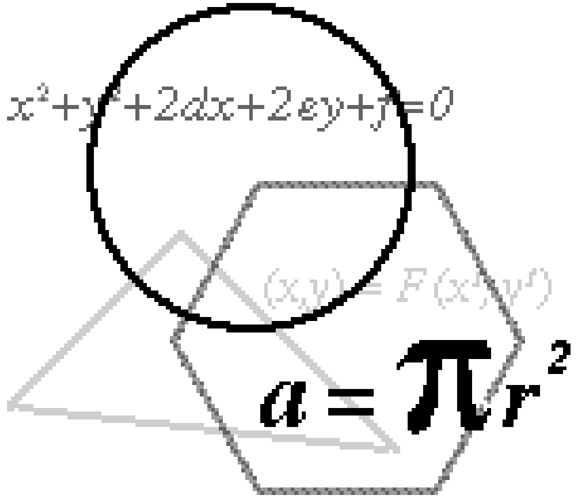
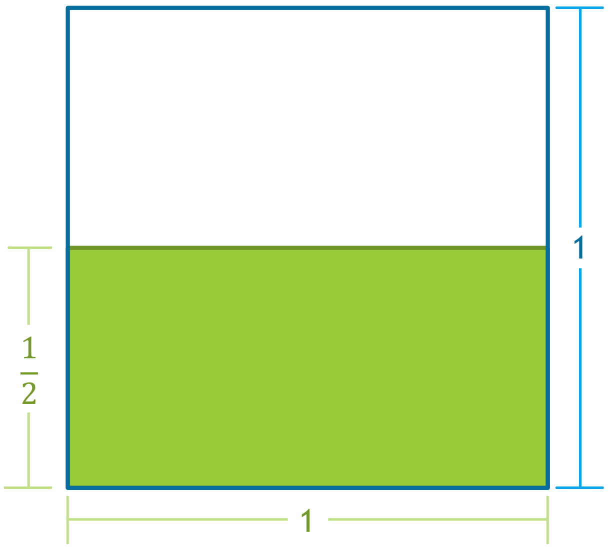
Formula-Free Geometry



Area and Volume

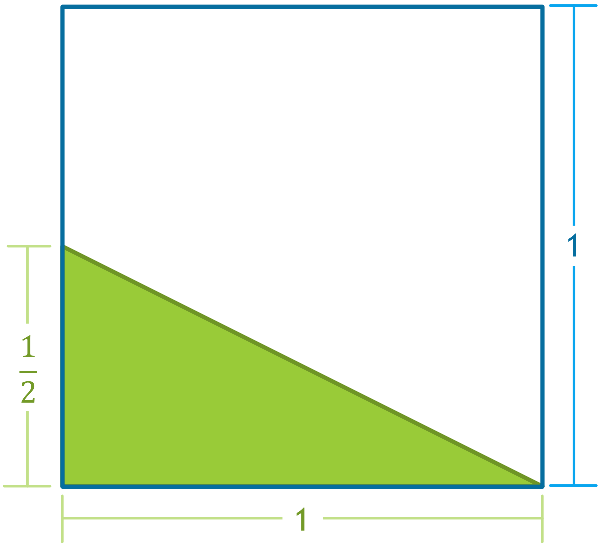
Exact Geometry

Easy!



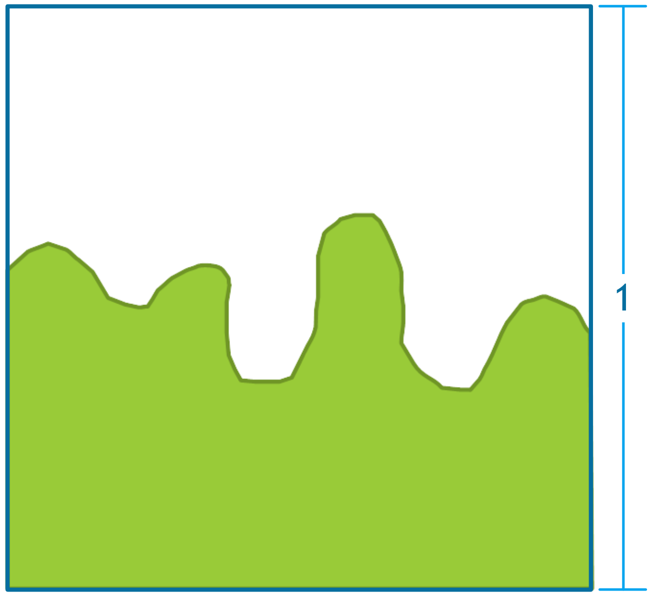
$A = bh$ = ½

Pretty easy!



$A = ½bh$ = ¼

Inexact Geometry



No formula for green area

$$A = ?$$

Random Procedure

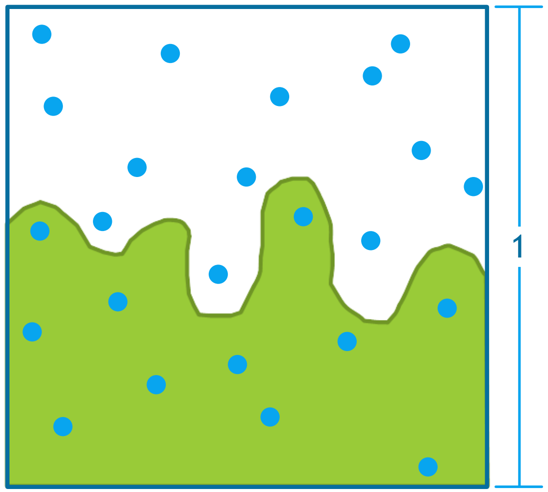
Uniform Random

Occurring with equal probability

- Drop a point at any location within the square
- The likelihood of a point falling inside the region is determined by the proportion of area that the shaded region fills

$$\frac{N_{in}}{N_{tot}} \approx \frac{A_{in}}{A_{tot}}$$

Random Points

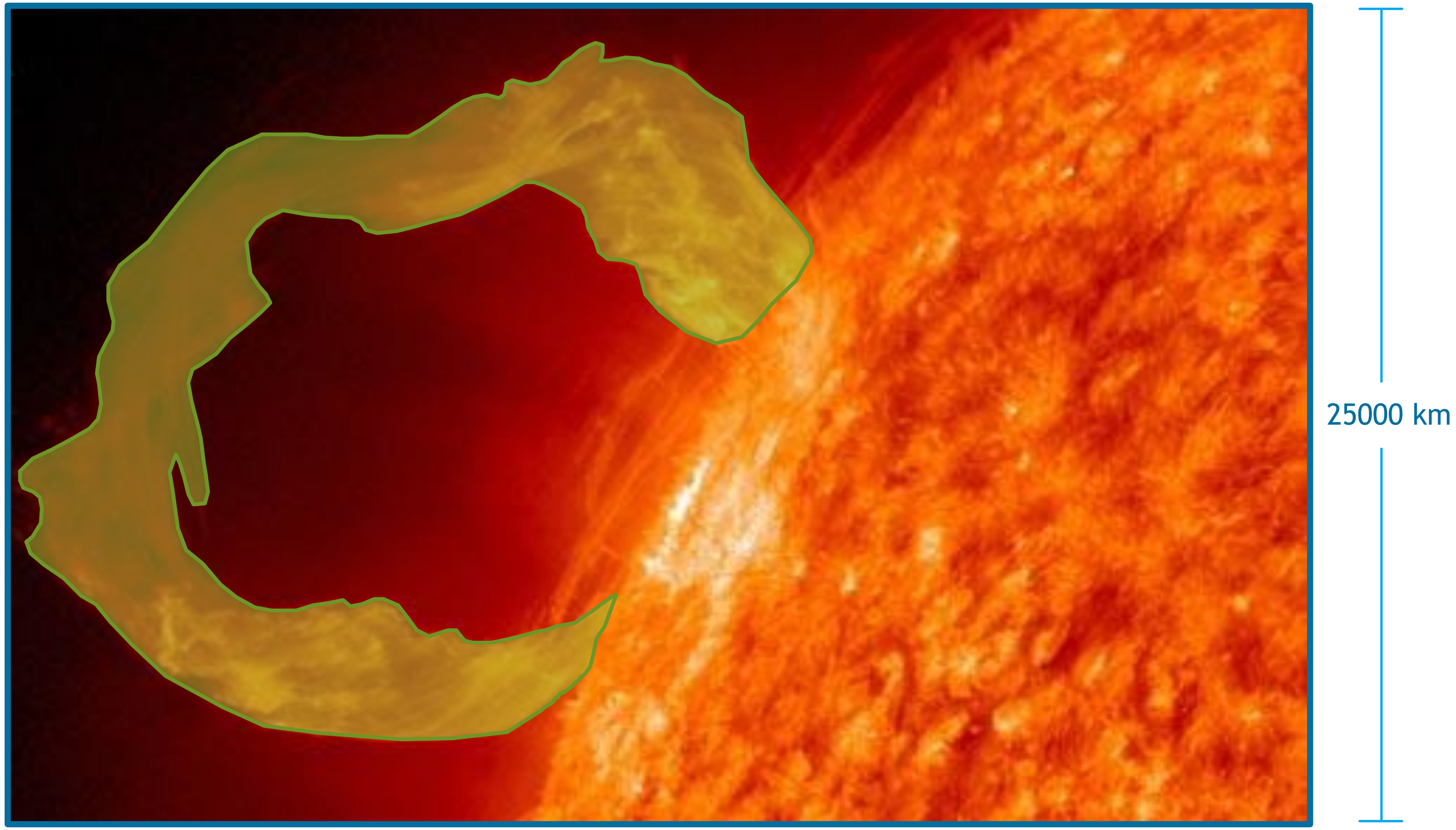


- 24 points total
- 11 points inside
- $\frac{N_{in}}{N_{tot}} \approx \frac{A_{in}}{A_{tot}}$

green area ≈ $^{11}/_{24}$

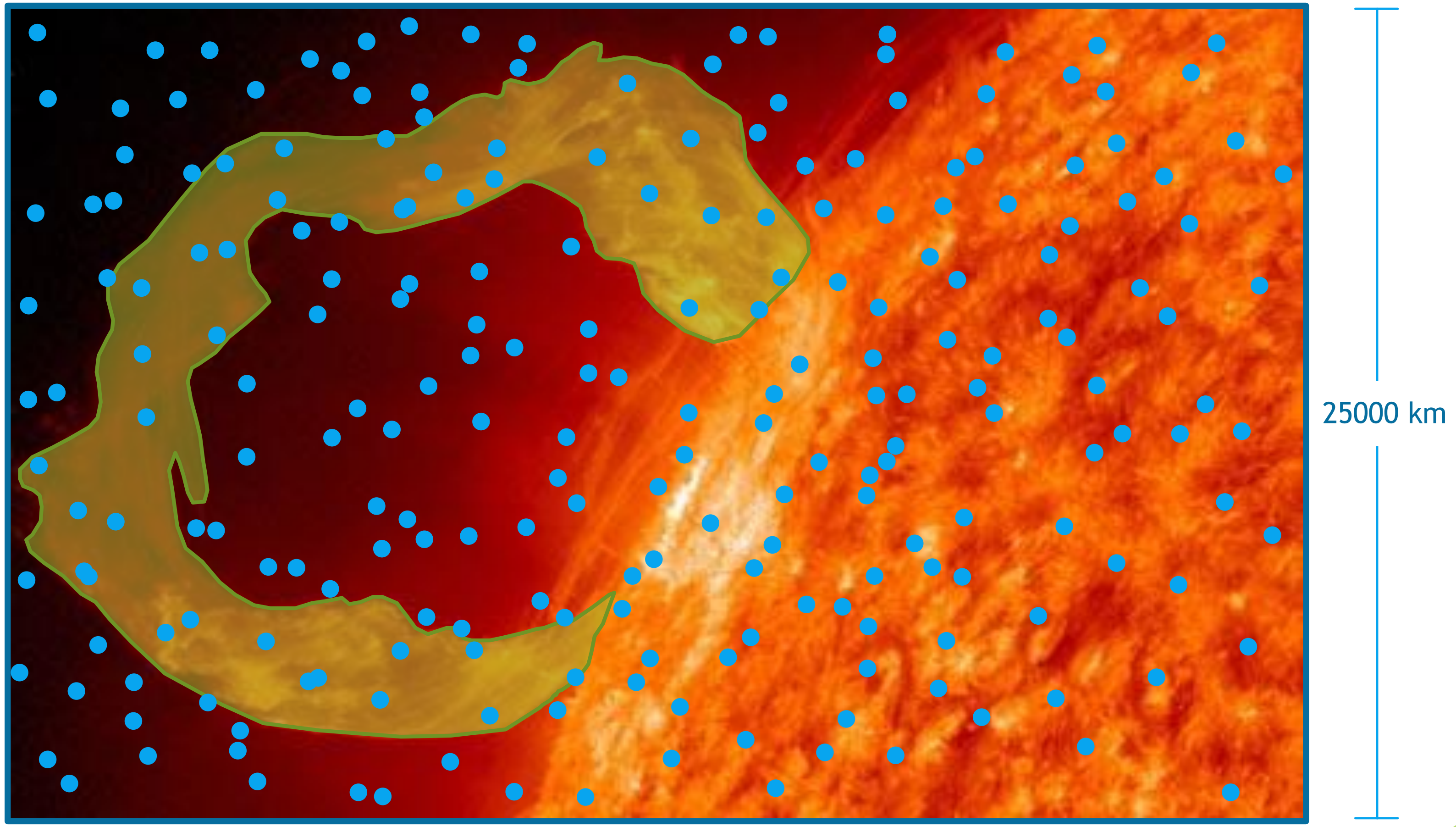
Real Geometry

Can we use this for real problems? YES!



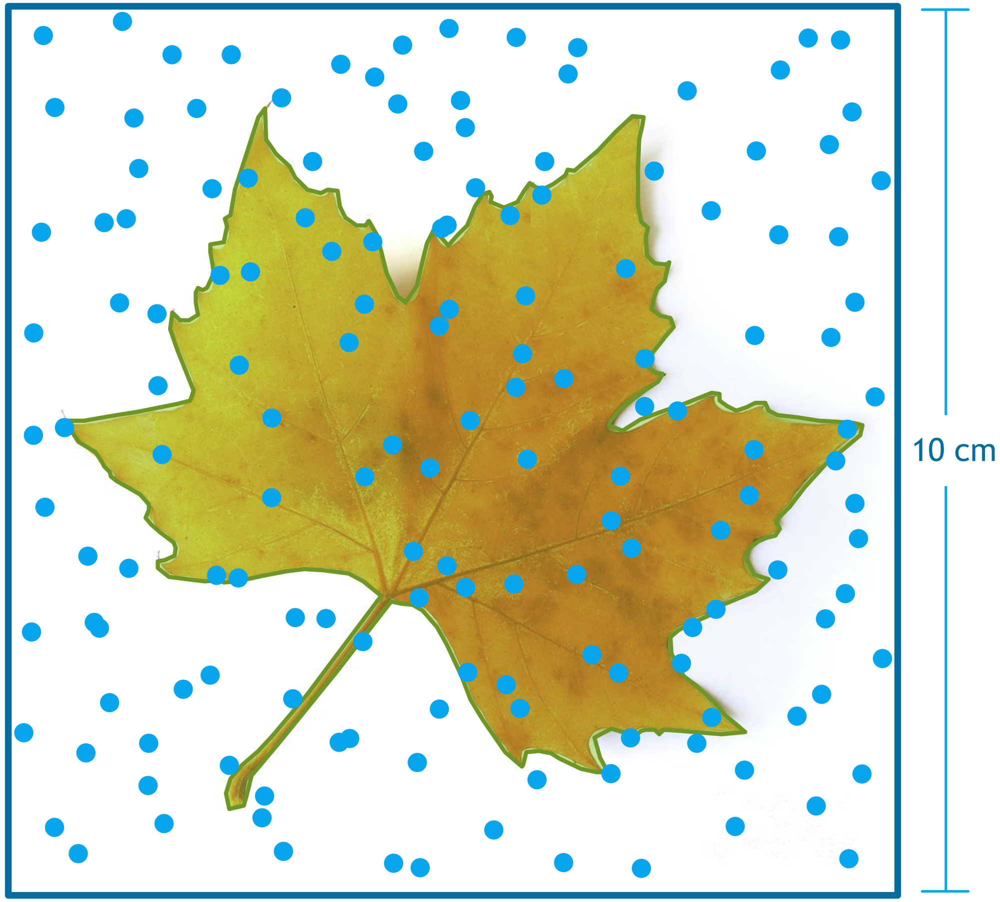
Real Geometry

Use a computer to generate MANY random points

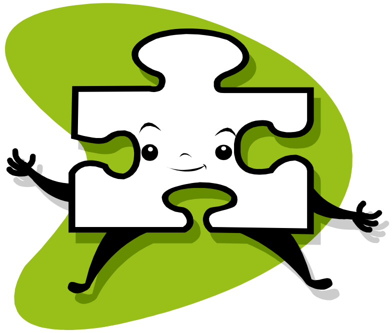


Real Geometry

Real Geometry



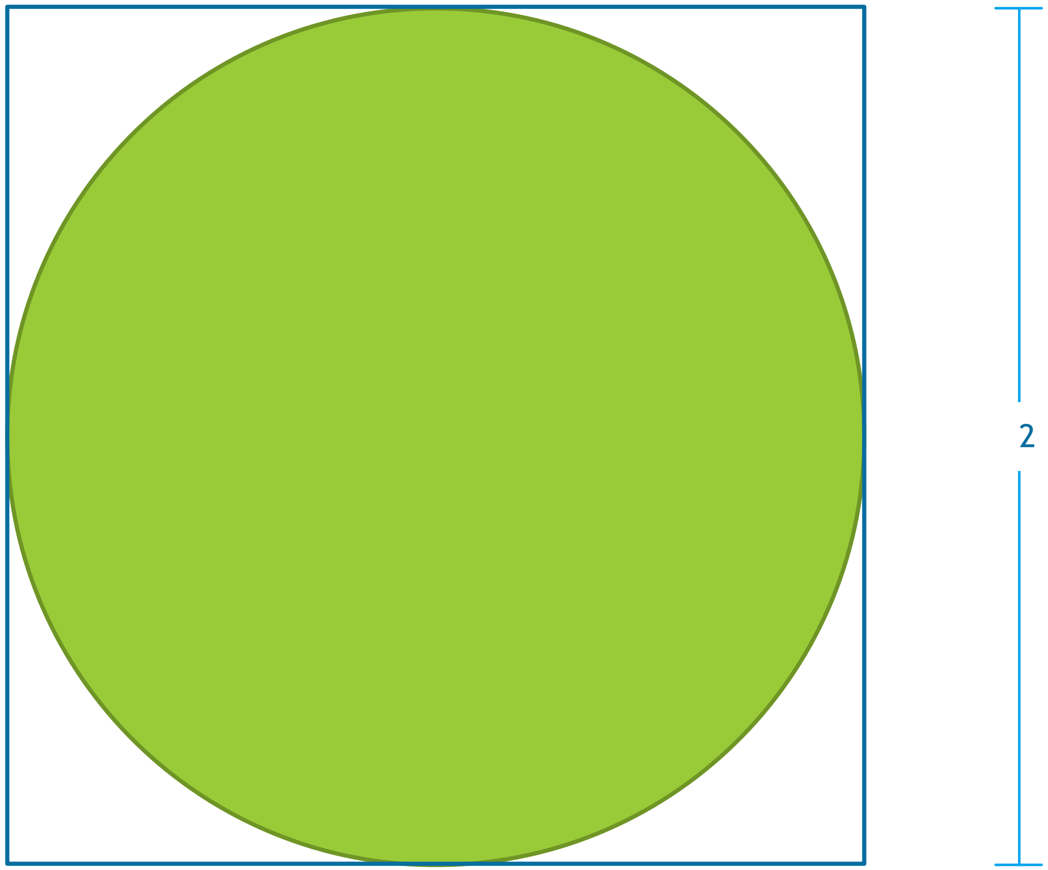
Today’s Challenge



- The constant π describes the geometry of any circle of any size.
- Ever wonder why π =3.14159?
- Today, we’ll use approximated geometry to investigate π

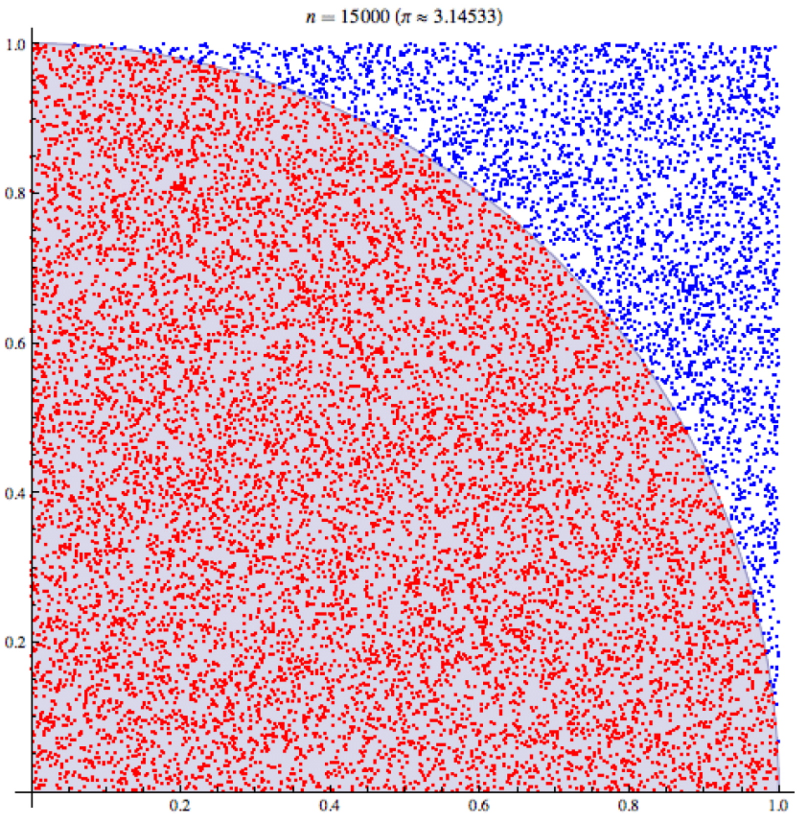
Circle in a Square

Known formula: if you know π



Simulated Random Points

Use the EV3 Brick to…



- Simulate many random 2-D points
- Use $x^2 + y^2 = r^2$
- Use $\frac{N_{in}}{N_{tot}}$ to estimate area

How Close to π?

- Find average of five estimates
- Find the percent error relative to 3.14159
- Find the ‘standard error’ of samples
  - Tighter error → better confidence

π

How Close? Standard Error

- Samples: {3.1635, 3.1393, 3.1453}
- Average: $\bar{x}$ = 3.1494
- Sum of deviations:
  - S = $(3.1635 - \bar{x})^2 + (3.1393 - \bar{x})^2 + (3.1453 - \bar{x})^2$
  - S = 0.0003176
- Standard Error:
  - SE = $x = \sqrt{\frac{S}{3(3-1)}}$
  - SE = .007276