**Notetaking Sheet**

*Free Body Diagrams*

By definition, a free-body diagram is a representation of an object with all the forces that act on it. The external environment, as well as the forces that the object exerts on other objects, are omitted in a free-body diagram. This allow us to analyze an object in isolation. A free-body diagram (FBD) can be constructed in three simple steps: first, sketch what is happening on the body; second, identify the forces that act on the object; and third, represent the object as a point, the forces as arrows pointing in their acting direction, with origin at the point representing the object, with a size proportional to their magnitude and a label indicating the force type.

Practice. A solid with weight equal 10 N is suspended from the ceiling with two ropes, as shown in the diagram. Use a Free Body Diagram to determine the value the tensions on the two ropes

*x*

*y*

60o

40o

10 N

*x*

*y*

*x*

*y*

**Bridge.** Structure that is built over a railroad, river, or road so that people or vehicles cross from one side to the other.

**Truss.** A regular structure or frame built with straight members with end point connections and forces that act only at these end points. No member is continuous through a joint.

**Truss Bridge.** The bridge whose load-bearing superstructure is composed of a truss, a structure of connected elements usually forming triangular units. The connected elements (typically straight) may be stressed from tension, compression, or sometimes both in response to dynamic loads. The basic types of truss bridges (11) are shown below:

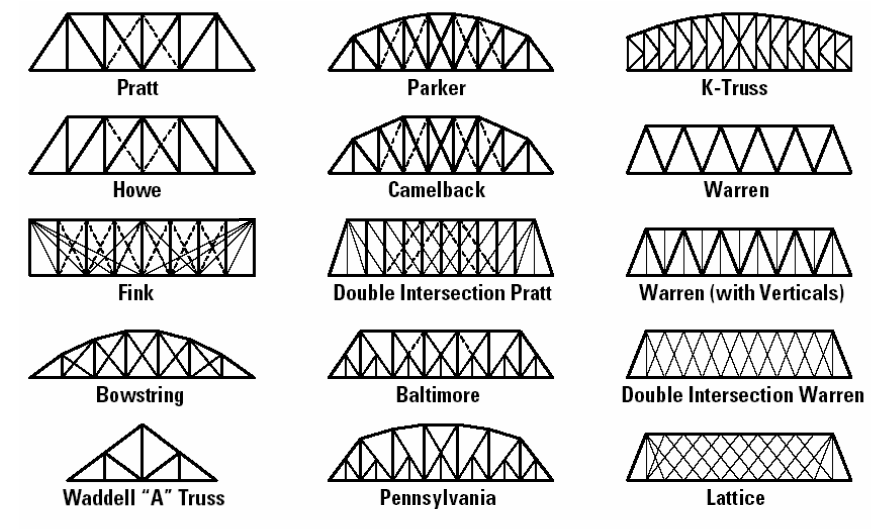


Fig. 01. Common types of Truss Bridges (11)

**Warren Truss.** Design distinguished by equal-sized members and the ability of some of the diagonals to act in both tension and compression. The type is generally characterized by thick, prominent, diagonal members, although verticals could be added for increased stiffness.

Warren truss

Warren truss w/verticals

**Pratt and Howe Trusses.** These trusses are very similar, their trusses’ elements are arranged in right triangles. They differ in the orientation of the hypotenuse of these triangles (Fig. 16). The analysis of forces on these trusses is very similar.

Howe

Pratt

*Warren Truss Analysis. The Method of Joints*

In this section will be solved the simplest truss bridge: the Warren. The bridge structure in this example will be also the smallest possible, only three triangular elements, equilateral triangles of 4 in sides, and five nodes (Fig.07). The procedure here detailed can be extended to larger trusses.

60o

4 in



2

1

3

4

5

Fig. 07. Warren truss bridge. Three equilateral triangles elements

In this analysis it is considered that (Fig.08):

(a). Vertical and equal downward forces of 10 lbf are applied on the top nodes 2, 4, and low 3: *F2, F3, F4*

(b). The bridge is supported only by reaction forces at bottom nodes 1 and 5: *R1, R5*

(c). Only tension and compression forces are considered acting along the structure’s segments (Fig,08): *Fij (Force acting between node i and node j)*

(d). Truss Elements are considered rigid. Structure’s segments do not bend.

(e). Once determined a tension or compression force at one end of the segment, the complementary force at the other end will be equal but in opposite direction: *Fij = - Fji* or *F21 = - F12*, *F31 = - F13*, *F32 = - F23*, etc.

5

*F24*

*F42*

*F23*

*F21*

*F43*

*F45*

*F2*

*F4*

*R1*

*F12*

*F34*

*F32*

2

4

*R5*

*F54*

3

1

*F3*

*F31*

*F13*

*F35*

*F53*

Fig.08. Vertical downward forces on upper truss nodes and bottom mid node. Reaction forces at end bottom nodes .

*Practice*: Write on the next diagram, the corresponding forces acting on each of the truss nodes, according with the assumptions (a)-(e)

*R1*

2

1

3

4

5

*Practice. Find the value of the Forces of Reaction on Nodes 1 and 5, using Momentum of Forces.*

*Assumption: Bridge does not move then the momentum of all the vertical forces has to be zero*

Momentum at Node 1

4

3

2

5

**1**

Momentum at Node 2

4

**5**

3

1

2

*Practice*: Analysis of Forces on Nodes using Free Body Diagrams and the assumptions ∑*Fy* = 0 and ∑*Fx* = 0

*x*

*y*

Node 1.

1

*x*

*y*

Node 2.

2

*x*

*y*

Node 3.

3

Node 4

*x*

*y*

4

*x*

*y*

Node 5

5

*Practice*: Put together the above obtained equations as a System of Equations. Label equations from (1) to (10)

|  |  |  |  |
| --- | --- | --- | --- |
| **Nodes** | 1 | ∑*Fy* = 0 |  |
| ∑*Fx* = 0 |
| 2 | ∑*Fy* = 0 |  |
| ∑*Fx* = 0 |
| 3 | ∑*Fy* = 0 |  |
| ∑*Fx* = 0 |
| 4 | ∑*Fy* = 0 |  |
| ∑*Fx* = 0 |
| 5 | ∑*Fy* = 0 |  |
| ∑*Fx* = 0 |

*Practice: Solve the System of Equations. Identify the obtained forces as Compression* (-) *or Tension* (+)

* *Solve equation* (1) *for F12:*
* *Substitute the obtained value for F12 in equation* (2)*, and solve for F13:*
* *In equation* (3) *substitute the value for F12, and solve for F23;*
* *In equation* (4) *substitute the values for F12 and F23, and solve for F24:*
* *In equation* (5) *substitute the value for F23 and solve for F34:*
* *In equation* (6) *substitute the values for F13, F23 and F34, and solve F35:*
* *In equation* (7) *use the value for F34 and solve for F45:*

*Practice*. Create the matrix for the above System of Equations

|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Node |  | *F12* | *F13* | *F23* | *F24* | *F34* | *F35* | *F45* |  |  |

|  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| 1 | *∑Fy* = 0 |  |  |  |  |  |  |  |  |  |  |
| *∑Fx* = 0 |  |  |  |  |  |  |  |  |  |  |
| 2 | *∑Fy* = 0 |  |  |  |  |  |  |  |  |  |  |
| *∑Fx* = 0 |  |  |  |  |  |  |  |  |  |  |
| 3 | *∑Fy* = 0 |  |  |  |  |  |  |  |  |  |  |
| *∑Fx* = 0 |  |  |  |  |  |  |  |  |  |  |
| 4 | *∑Fy* = 0 |  |  |  |  |  |  |  |  |  |  |
| *∑Fx* = 0 |  |  |  |  |  |  |  |  |  |  |
| 5 | *∑Fy* = 0 |  |  |  |  |  |  |  |  |  |  |
| *∑Fx* = 0 |  |  |  |  |  |  |  |  |  |  |

*Graphing Interface to Calculate Tensions-Compressions on Truss Bridges*

This section summarizes how to use a very friendly computation interface developed in Google Sheets. This Graphic Interface:

* Calculates the tensions-compressions on the straight elements in the Warren Truss, Warren with Vertical Truss, Pratt Truss, and Howe Truss.
* Estimates the maximum strength of the trusses considering kind of wood used and element’s thickness.
* Gives only the solution when the truss is supported on its bottom end nodes only, the truss diagonal elements are round, and the truss rails are square. For square diagonals enter a diameter value such that the resulting cross sections matches the cross section area of the square diagonal.

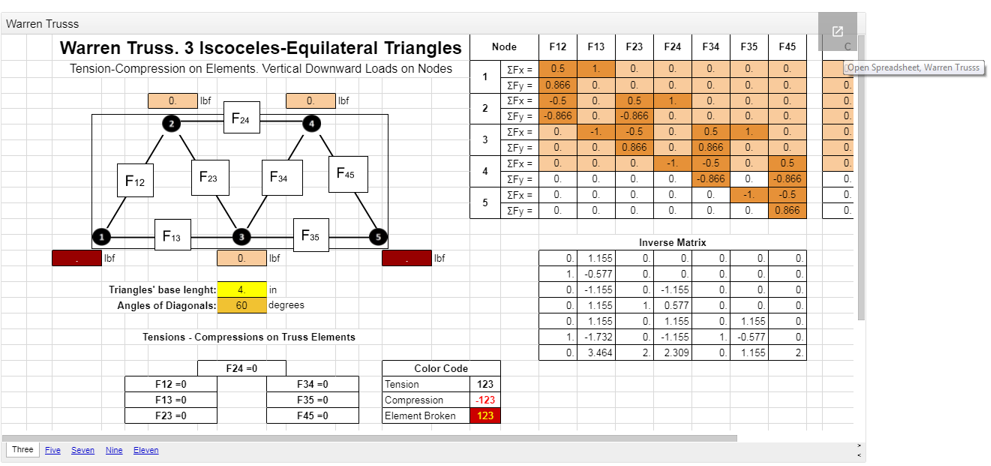
The link to these interfaces is:

<https://sites.google.com/gpapps.galenaparkisd.com/mramirez-math/courses-highlights/bridges/trusses-calculations>

In this web page there are four interfaces (Figure 14(a)) corresponding to the four different trusses. Truss size is defined here by the number of triangles in the truss. Next table summarizes truss types and sizes:

|  |  |
| --- | --- |
| **Truss Type** | **Size (No. of triangles)** |
| Warren | 3, 5, 7, 9, 11 (equilateral-isosceles) |
| Warren w/ Verticals | 3, 5, 7, 9, 11 (equilateral-isosceles) |
| Pratt | 6, 10,14 (right triangles) |
| Howe | 6,10,14 (right triangles) |

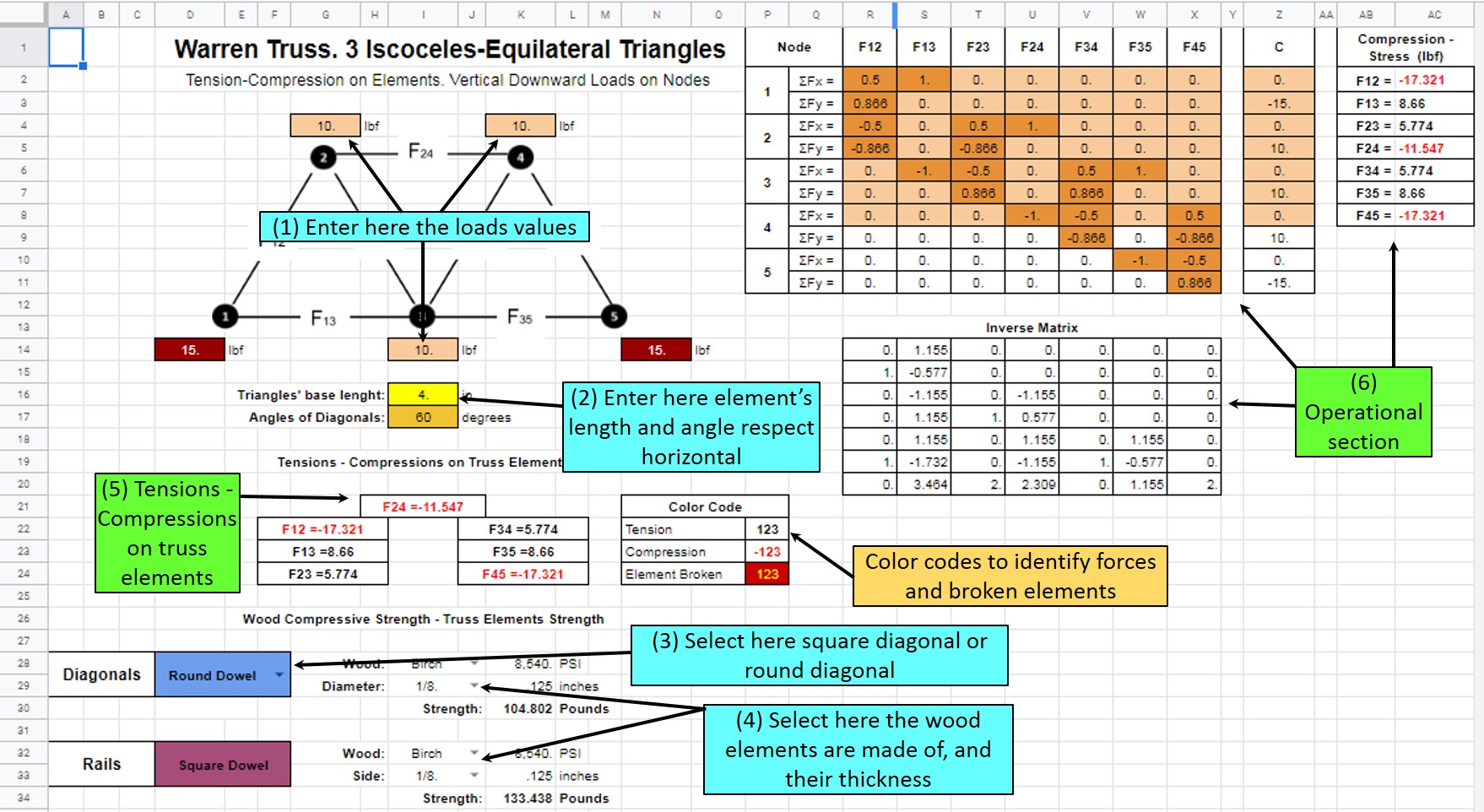
To activate an interface in PC’s, laptops, and tablets, click on the gray square at the right top of the window. Click on the window when using a cellphone.



These interfaces are shared to everybody in the web, so any user opening these documents can input information. These worksheets are partially protected, such that only specific cells can be edited. This prevents a user accidentally or intentionally deletes important working formulas.

Once the spreadsheet is active, you can see its different sections

1. A truss diagram with entries for the loads on each node.
2. Two entries for the truss elements’ length and angle respect the horizontal.
3. Cells to select the truss elements’ thickness and kind of wood they are made of.

**

1. Section displaying the calculated truss elements’ tensions-compressions
2. The operational section: matrix associated to the system of linear equations obtained from the FBD’s, the inverse matrix, and the solutions of the system of equations.

You only have to input values in the cells in sections (1) and (2), and select the elements’ wood type and thickness in the cells in section (3). Sections (4) and (5), or other cells **do not have to be altered**. Tensions-Compressions on the truss elements are automatically calculated once values in sections (1), (2), or (3) are entered.

**Important.**

Because this files are open to everybody, everybody can modify your work and you can modify everybody’s work. So to keep your work’s integrity you will have to save a copy in your Google Drive of the interface you have to use.

*Use the commands* Share & export + Make a copy*, give a new name to the file and save it in your* Drive*. You will work now in your saved file.*

**WARNING.**Even though the original file cells that do not have to be modified are protected, the copies you saved possibly will not be protected. BE VERY CAREFUL and do not modify or erase other cells but the indicated in the practices.

*Practice. Use the Google Slides Graphic Interfaces to solve four of the next problems. Your teacher may assign some specific to you.*

1. *Find the minimum Diagonals and Rails thicknesses for a Hardwood Warren Truss* 20*in long, made up with nine equilateral triangles, able to resist a load of* 50 *lfb on every top node and* 100 *lbf on every bottom node*

2. *Find the minimum Diagonals and Rails thicknesses for a Hardwood Pratt Truss* 20*in long, made up with six* 30-60-90 *right triangles, able to resist a load of* 50 *lfb on every top and bottom node.*

3. *Find the minimum Diagonals and Rails thicknesses for a White Pine Warren Truss* 24*in long, made up with seven equilateral triangles, able to resist a load of* 60 *lfb on every top and bottom node.*

4. *Find the minimum Diagonals and Rails thicknesses for a Basswood Pratt Truss* 20*in long, made up with six* 30-60-90 *right triangles, able to resist a load of* 50 *lfb on every top and bottom node.*

*5. Find the minimum Diagonals and Rails thicknesses for a Hardwood Warren Truss w/verticals,* 20*in long, made up with five equilateral triangles, able to resist a load of* 70 *lfb on every top and 80 lbf on every bottom node.*

6. *Find the minimum Diagonals and Rails thicknesses for a Hardwood Pratt Truss* 20*in long, made up with ten* 30-60-90 *right triangles that be able to resist a load of* 60 *lfb on every top node and* 80 *lbf on every bottom node*

7. *Find the proper wood to build a Howe Truss* 36*in long, made up with fourteen* 30-60-90 *right triangles,* 3/8*in Diagonals,* 3/8*in Rails, and able to resist a load of* 40 *lfb on every top and bottom node.*

*8. Find the proper wood to build a Pratt Truss* 32*in long, made up with fourteen* 30-60-90 *right triangles, 1/4in Diagonals,* 5/16*in Rails, and able to resist a load of* 60 *lfb on every top node and bottom node.*

9. *Find the proper wood to build a Warren Truss* 36*in long, made up with 11 equilateral triangles, 5/16in Diagonals,* 1/2*in Rails, and able to resist a load of* 80 *lfb on every top node and bottom node.*

10. *Find the proper wood to build a Howe Truss* 30*in long, made up with ten* 30-60-90 *right triangles, 5/16in Diagonals,* 5/16*in Rails, and able to resist a load of* 50 *lfb on every top node and bottom node.*

11. *Find the proper wood to build a Warren Truss* 30*in long, made up with nine equilateral triangles,* 1/4*in Diagonals,* 5/16*in Rails, and able to resist a load of* 50 *lfb on every top node and bottom node.*

12. *Find the proper wood to build a Pratt Truss* 30*in long, made up with fourteen* 30-60-90 *right triangles*, 5/16*in Diagonals,* 1/2*in Rails, and able to resist a load of* 60 *lfb on every top node and* 80 *lbf on every bottom node.*

13*. Find the maximum load equally distributed on every top node and on every bottom node a Poplar Howe Truss* 30*in long, made up with ten* 30-60-90 *right triangles, is able to resist, having* 1/4*in diagonals and* 3/8*in rails .*

14*. Find the maximum load equally distributed on every top node and on every bottom node a White Pine Howe Truss* 36*in long, made up with fourteen* 30-60-90 *right triangles, is able to resist, having* 5/16*in diagonals and* 5/16*in rails .*

15*. Find the maximum load equally distributed on every top node and on every bottom node a Poplar Pratt Truss* 36*in long, made up with ten* 30-60-90 *right triangles, is able to resist, having* 1/8*in diagonals and* 3/8*in rails .*

16*. Find the maximum load equally distributed on every top node and on every bottom node a White Pine Pratt Truss* 34*in long, made up with fourteen* 30-60-90 *right triangles, is able to resist, having* 3/8*in diagonals and* 5/16*in rails .*

17*. Find the maximum load equally distributed on every top node and on every bottom node a Poplar Warren Truss* 30*in long, made up with eleven equilateral triangles, is able to resist, having* 3/8*in diagonals and* 3/8*in rails .*

18*. Find the maximum load equally distributed on every top node and on every bottom node a Hardwood Warren Truss w/Verticals* 36*in long, made up with nine equilateral triangles, is able to resist, having* 1/4 *in diagonals and* 3/8 *in rails .*

***Note***: *Use the format at the back of this page to write your answers.*

*Problem \_\_\_\_\_\_\_\_*

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| **Truss** | **# Triangles in Truss** | **Span (in)** | **Diagonals/ Verticals Shape** | **Diagonal/ Verticals Thickness (in)** | **Rails Thickness (in)** | **Wood** | **Total/ Maximum Load (lbf)** |
|  |  |  |  |  |  |  |  |

*Problem \_\_\_\_\_\_\_\_*

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| **Truss** | **# Triangles in Truss** | **Span (in)** | **Diagonals/ Verticals Shape** | **Diagonal/ Verticals Thickness (in)** | **Rails Thickness (in)** | **Wood** | **Total/ Maximum Load (lbf)** |
|  |  |  |  |  |  |  |  |

*Problem \_\_\_\_\_\_\_\_*

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| **Truss** | **# Triangles in Truss** | **Span (in)** | **Diagonals/ Verticals Shape** | **Diagonal/ Verticals Thickness (in)** | **Rails Thickness (in)** | **Wood** | **Total/ Maximum Load (lbf)** |
|  |  |  |  |  |  |  |  |

*Problem \_\_\_\_\_\_\_\_*

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| **Truss** | **# Triangles in Truss** | **Span (in)** | **Diagonals/ Verticals Shape** | **Diagonal/ Verticals Thickness (in)** | **Rails Thickness (in)** | **Wood** | **Total/ Maximum Load (lbf)** |
|  |  |  |  |  |  |  |  |