Name: Date: Class:

Tower Height, Surface Area and Volume Quiz

Assume we are using six sheets of A3 (279 mm x 420 mm) cardstock to build our tower. Recall from our investigation of the tower that we found that the volumes and surface areas of successive blocks to be in geometric progressions.

1. Fill in the common ratios
   a. Common ratio for block volumes: $r_v =$ _____
   b. Common ratio for block surface areas: $r_a =$ _____

2. Now consider the block heights. Notice that these are also in a geometric progression. Find the common ratio:

   $r_h =$ _____

3. It turns out the geometric progressions can be summed up when the common ratio is between zero and one. This problem will guide you through the process of understanding this concept.
   a. Let v be the volume of the first block. $v =$ _____
   b. Given the volume v of the first block and the common ratio r_v we can find the volume of the entire tower V_T like this:

$$V_T = v + v \cdot (r_v) + v \cdot (r_v)^2 + v \cdot (r_v)^3 + \cdots$$
$$= v \cdot (1 + r_v + r_v^2 + r_v^3 + \cdots)$$

The trick is to find the value of the sum of the powers of the common ratio r_v. Here's the trick. Let $1 + r_v + r_v^2 + r_v^3 + \cdots = s$. We can subtract 1 from both sides to get:

$$r_v + r_v^2 + r_v^3 + \cdots = s - 1$$

Then we factor out the common ratio on the left-hand side:

$$r_v \cdot (1 + r_v + r_v^2 + r_v^3 + \cdots) = s - 1$$

Notice that the sum in the parentheses is the original sum s!

Substitute the s in to the equation and solve for it.

$s =$ _____

c. Now we can find the volume of the tower!

$V_T =$ _____

4. a. Repeat the process in problem (3) to find the surface area A_T of the whole tower. Recall that the bottom of the blocks are not part of the surface area nor is the section of the top that the subsequent block rests on!

b. In the project we learned where to position the elevator shaft so that we could reach any floor. Now determine how tall the elevator shaft would need to be to reach them.

5. The tallest building in the world (as of May 2020) is the Burj Khalifa in Dubai, UAE topping out at an impressive 2,722 ft (828 m) which is over half a mile high! You're hired by a developer to design a tower like we made with cardstock that is to be 3,000 ft (914 m) high, so it will replace the Burj Khalifa as the tallest building in the world.

   a. Realistically the elevator shaft will only go as high as the smallest inhabitable room, the penthouse. We define inhabitable to mean having a ceiling that is at least 8 ft (2.44 m) high. Which floor will the elevator shaft reach under these conditions?

   b. Assuming the surface will be mostly glass and that this glass costs about $100 per square foot, how much will all the glass cost to make the tower?

   c. How much real estate does this tower require? In other words, what is the building footprint at the ground level in square feet?